SRI AKILANDESWARI WOMEN'S COLLEGE, WANDIWASH

SET THEORY

Class : I PG MATHEMATICS

Mrs.P. Kavitha
Assistant Professor
Department of Mathematics

SWAMY ABEDHANADHA EDUCATIONAL TRUST, WANDIWASH

INTRODUCTION

Set Theory is the branch of mathematical logic that studies set, which can be informally described as collection of objects.Although objects of any kind can be collected into a set,Set theory assignment a branch of mathematics is mostly concerned with those that are relevant to mathematics as a whole.Set theory was developed by German mathematician Georg Cantor (1845-1918).Set theory is a foundation for a better understanding of topology, abstract algebra and discrete mathematics.Understanding set theory will also help in understanding other mathematical concepts like relation, function, probability,etc.

BASIC DEFINITIONS

SET:

In mathematics,a set is defined as a collection of distinct,well-defined objects forming a group. There can be any number of items,It is collection of whole numbers,monthsof a year ,types of birds,and so on .Each item in the set is known as an element of the set

EXAMPLE:

- Set of Natural Numbers: N={1,2,3,4.........}
- Set of Even Numbers:E={2,4,6,8........}

REPRESENTATION OF SET

ROSTER FORM

- This form is also called Tabular form.
- In roster form all the elements of a set are listed,the element are being separated by commas and are enclosed within braces{ }

EXAMPLE:

If A is the set of all prime numbers less than 11,then

$A = \{2,3,5,7\}$

SET BUILDER FORM

- This form is also called property form.
- In set builder form, a rule or a statement describing the common characteristic of all the elements is written instead of writing the elements directly inside the braces

EXAMPLE :

If A is set of all even natural numbers,then

$A = \{x : x \in N, x = 2n, n \in N\}$

TYPES OF SET

- empty set
- Singleton set
- Finite set
- Infinite set
- power set
- Universal set
- Equivalent set
- Disjoint set
- Equal set
- Overlapping set

- **Empty Set:**

  A set that contains no element is called empty set or null set.

  It is denoted by { } or Ø

- **Singleton set:**

  A set which has only one element is called singleton set .

  Example : 1. A={3}

  2. B= {x:4<x<6 and x is an integer}

- **Finite set :**

  A set is finite if it consists of a definite number of different elements.

  Example : A= The set of all schools in Pakistan

- **Infinite set:**

A set which having a uncountable number of elements is called infinite set.

Example: N = A set of all natural numbers.

- **Power set:**

The set of all subset of a set A is called the power set of 'A'. It is denoted by P(A).

Example: A={1,2,3}

P(A)={Ø,{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}.

- **Universal set:**

A set which contains all the elements of all the sets under consideration and is usually denoted by U

Example: U={2,3,4,5,6,7,8,9,10}

- **Equal set:**

Two sets are said to be equal if they Contain exactly the same elements ,otherwise they are said to be unequal.

Example: A={1,2,3,4}

B={4,3,2,1}

- **Overlapping set:**

If atleast any one element of the two sets are same then the two set are said to be overlapping sets.

Example:A={1,2,3}

B={3,4,5}

SUBSET

Let A and B are two sets.If every element of A is an element of B then $A \subseteq B$

Example: A={2,3} and B={1,2,3,4,5}

$A \subseteq B$

- **Proper subset:**

Let A and B be two sets. If A is a subset of B and $A \neq B$, then A is called a proper subset of B and we write $A \subset B$.

Example:If A = {1,2,5} and B = {1,2,3,4,5} then A is a proper subset of B. i.e. $A \subset B$.

- **Improper subset:**

A subset which contains all the elements of the original set is called an improper subset. It is denoted by $\subseteq$.

Example:

If set A ={1, 2, 3, 4} and B ={1, 2, 3, 4} then A is the improper subset of set B. It is mathematically expressed as A ⊆ B.

- **Singleton set:**

Singleton set is a set containing only one element. The singleton set is of the form A ={a}, and it is also called a unit set.The singleton set has two subsets, which is the null set, and set itself.

Example:

The given set is A ={1,3,5,7,11}. The given set has 5 element and it has 5 subsets which can have only one element and are singleton set. Therefore, the five singleton sets which are subsets of the given set A is {1}, {3}, {5}, {7}, {11}.

DE MORGAN 'S LAW

De Morgan's Law is a collection of Boolean algebra transformation rules that are to connect the intersection and union of sets using complements. De Morgan's Law states that two conditions must be met. These conditions are typically used to simplify complex expressions. This makes performing calculations and solving complications and complicated Boolean expressions easier.

De Morgan's Law states that the complement of the union of two sets is the intersection of their complements, and also, the complement of two sets is the union of their complements.

. Depending on the inter-relation between the set-union and set-intersection, thereare two types of De Morgan's Law that exists in set theory. They are explained below:

- **First de Morgan's law**
- **Second de Morgan's law**

First De Morgan's law:

"The complement of the union of two sets is equal to the intersection of the complements of **each** set."

$$(A \cup B)' = A' \cap B'$$

Second De Morgan's law:

"The complement of intersection of two sets is equal to the union of the complements of each set."

$$(A \cap B)' = A' \cup B'$$

De Morgan's law for difference:

$$A - (B \cup C) = (A - B) \cap (A - C)$$

$$A - (B \cap C) = (A - B) \cup (A - C)$$

PROPERTIES OF SET

Properties of sets help in easily performing numerous operations across sets. The operation of union sets, intersection of sets, complement of set can be easily performed with the help of their respective properties. Many of the properties such as commutative property, associative property are similar to the properties of real numbers.

BASIC PROPERTIES OF SETS

1. Commutative Property

Intersection and union of sets satisfy the commutative property.

$A \cap B = B \cap A$

$A \cup B = B \cup A$

2. Associative Property

Intersection and union of sets satisfy the associative property.

$(A \cap B) \cap C = A \cup (B \cap C)$

$(A \cup B) \cup C = A \cup (B \cup C)$

3. Distributive Property

Intersection and union of sets satisfy the distributive property.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. Identity Property

Intersection and union of sets satisfy the identity property.

$A \cup \emptyset = A$

$A \cap U = A$

5. Complement Property

Intersection and union of sets satisfy the complement property.

$(A \cup A)' = U$

$(A \cap A)' = \emptyset$

6. Idempotent Property

Intersection and union of sets satisfy the idempotent property.

$A \cap A = A$

$A \cup A = A$

PARTITION

A partition of a positive integer n is a way to write n as a sum of positive integers.

Example:

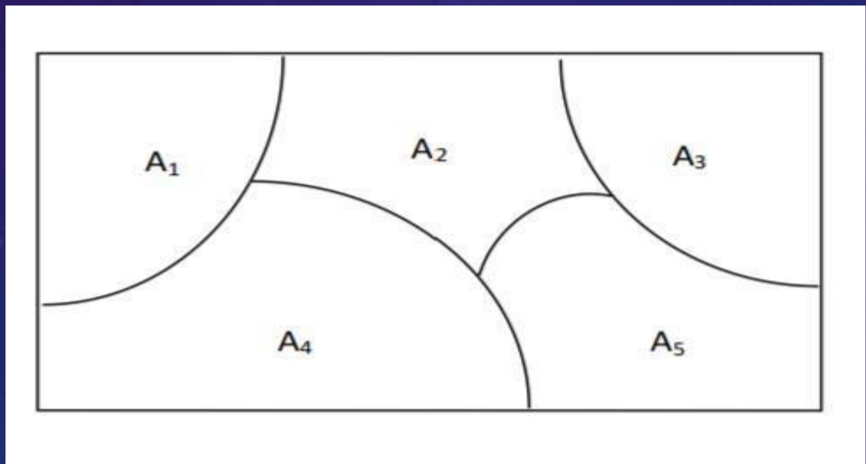
7 =3+2+1+1 is a partition of 7. Let Pm equal the number of different partitions of m, where the order of terms in the sum does not matter, and let Pm,n be the number of different ways to express m as the sum of positive integer not exceeding n.

Partition of set:

If S is a non-empty set, a collection of disjoint non-empty subsets of S whose union is S is called a partition of S. In other words, the collection of subsets A i is a partition of S if and only if.

1. A i ≠ Ø for every I.
2. A i∩A j = Ø when i ≠j and

3.UiA i =S, where Ui Ai represents the union of the subsets Ai for alI.



Example:

1. Let S ={1,2,3,4,5,6}

The collection of sets A1 ={1,2,3}, A2 ={4,5} and A3 ={6} forms a partition of S, since these sets are disjoint and their union is S.

2. The collection of sets A1 ={1,3,5}, A2 ={2,4} are not a partition, since the union of the subsets is not S, as the elements 6 is missing.

PRINCIPLE OF INCLUSION AND EXCLUSION

It status that number of elements in set operations can be calculated by counting elements which are not counted already (including them) and excluding (not counting) elements which are already added. This prevents double counting.

Consider two finite sets A and B. We can denote the principle of inclusion and exclusion formula as follows.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Here, n(A) denotes the cardinality of set A, n(b) denotes the cardinality of set B and $n(A \cap B)$ denotes the cardinality of $(A \cap B)$.

If we have 3 sets A, B and C then according to the principle of inclusion and exclusion ,

$$n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C).$$

In general,

$$n(A1 \cup A2 \cup A3 \cup .... \cup An) = n(A i \cap A j) + n(A i \cap A j \cap A k) + .. + (-1)n-1 \cap (A1 \cap A2 \cap A3 \cap .... \cap An)$$

MATHEMATICAL INDUCTION

- which we would use to prove any mathematical statement is ‘Principle of Mathematical Induction‘.

Mathematical Induction is a technique of proving a statement, theorem or formula which is thought to be true, for each and every natural number n. By generalizing this in form of a principle

- **ALGORITHMS**:

- Consider a statement P(n), where n is a natural number. Then to determine the validity of P(n) for every n, use the following principle:

- **Step 1:** Check whether the given statement is true for n = 1.
- **Step 2:** Assume that given statement P(n) is also true for n = k, where k is any positive integer.

Step 3: Prove that the result is true for P(k+1) for any positive integer k. If the above-mentioned conditions are satisfied, then it can be concluded that P(n) is true for all n natural numbers.

Proof:

The first step of the principle is a factual statement and the second step is a conditional one. According to this if the given statement is true for some positive integer k only then it can be concluded that the statement P(n) is valid for n = k + 1.

APPLICATION OF SET THEORY :

Analysis is an area of mathematics in which differential and integral calculus are essential components. Set theory is used to get a grasp of limit points and what is meant by the continuity of a function in this branch of mathematics.

- **Computer Science:** It's used in database systems, algorithms, and the design of computer languages, helping to organize and manipulate data efficiently.
- **Probability and Statistics:** It helps in defining sample spaces and eventsSet theory provides the basis of topology, the study of sets together with the properties of various collections of subsets.
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- It serves as the foundation for many mathematical subfields. It is used extensively in statistics, particularly in probability.

CONCLUSION

In this paper we learned about sets. Their definition and the different operations. Set theory is fundamental for many other branches like algebra and probability. Sets are a fundamental concept in mathematics with broad application across various field. We've explored the basic elements of sets, operations, and venn diagram, which are essential tools for defining relationship and categories, understanding sets not only lays a strong foundation in mathematics but also plays a crucial role in fields like statistics, computer science, and decision making. As you continue your mathematical journey, remember that sets are just the beginning of a fascinating world of abstract concept and logic. Mathematics is a vast and ever-evolving field, and your journey has only just begin.

THANK YOU